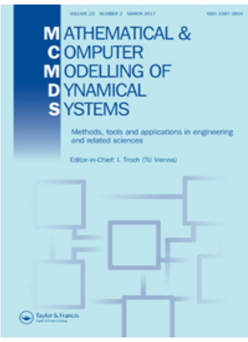

Taylor & Francis
Taylor & Francis Group

Mathematical and Computer Modelling of Dynamical Systems

Methods, Tools and Applications in Engineering and Related Sciences

ISSN: 1387-3954 (Print) 1744-5051 (Online) Journal homepage: http://www.tandfonline.com/loi/nmcm20

Fuzzy model predictive control of a DC-DC boost converter based on non-linear model identification

Robert Baždarić, Drago Matko, Aleš Leban, Danjel Vončina & Igor Škrjanc

To cite this article: Robert Baždarić, Drago Matko, Aleš Leban, Danjel Vončina & Igor Škrjanc (2017) Fuzzy model predictive control of a DC-DC boost converter based on non-linear model identification, Mathematical and Computer Modelling of Dynamical Systems, 23:2, 116-134, DOI: 10.1080/13873954.2016.1232283

To link to this article: https://doi.org/10.1080/13873954.2016.1232283

Published online: 20 Sep 2016.

MATHEMATICAL AND COMPUTER MODELLING OF DYNAMICAL SYSTEMS, 2017
VOL. 23, NO. 2, 116–134
http://dx.doi.org/10.1080/13873954.2016.1232283

Fuzzy model predictive control of a DC-DC boost converter based on non-linear model identification

Robert Baždarić, Drago Matko, Aleš Leban, Danjel Vončina and Igor Škrjanc

Faculty of Electrical Engineering, University of Ljubljana, Ljubljana, Slovenia

ABSTRACT

We present a novel method for the fuzzy control of a DC-DC boost converter based on a new approach to modelling the converter using Takagi–Sugeno (T-S) fuzzy identification. Two grades of identification result in a *global model* of a non-linear dynamical system and its finite impulse response model (FIRM) expression, which is therefore applicable in various model predictive control (MPC) standard methods with constraints. The successful simulation and experimental results shown in this study indicate the robustness and demonstrate stable operation of the DC-DC converter, even in the dynamic exchange of the discontinuous conduction mode (DCM) and the continuous conduction mode (CCM) with the preservation of a similar transient time. Although the study was primarily conducted on a hybrid simulation model of the DC-DC boost converter, the presented method is insensitive to the complexity of the physical process, as it suggests identified model-based control and emphasizes a new, general approach to pulse energy converter (PEC) controls. The statement is pursued with the subsequent application to the physical system of the converter. Furthermore, it underlines the method's consideration of real-time processing and its final online simplicity.

ARTICLE HISTORY
Received 10 February 2016
Accepted 31 August 2016

KEYWORDS
DC-DC boost converter; hybrid modelling; fuzzy identification; robust control of non-linear dynamical systems; fuzzy dynamic matrix control (FDMC); fuzzy predictive functional control (FPFC)

1. Introduction

Hybrid dynamical systems are scientifically very interesting because of their presence in modern technology. System control is one of the leading disciplines that have been proven it in the last 20 years and have contributed to the growth of the cyber physical world around us. A DC-DC boost converter is a member of the switched affine systems (SAS), part of the subgroup of hybrid dynamical systems [1,2]. Its importance cannot be seen just through an exemplification in the theoretical sense, but it is also still an up-to-date solution in the field of pulse energy converters (PECs).

The hybrid structure [2,3], robust solution [3,4], natural constraints [3], complexity reduction [5,6] and emerging problem of non-linearities exclusion [7–10], can all be recognized as appealing tasks for the control of a DC-DC boost converter. The state-of-the-art control solutions [3] are mostly based on linear matrix inequalities (LMIs) optimizations in hybrid systems [3,5,6], relaxation algorithms in a sense of complexity reduction [11], complementarity formalism in reducing the modelling ambiguity [12,13], sliding mode control [14,15] and heuristic approaches, neural networks and fuzzy controls [10,16]. This study, as a continuation of previous results [16] in considering all of the objectives mentioned above, is suggesting and

CONTACT Robert Baždarić robert.bazdaric@fe.uni-lj.si Faculty of Electrical Engineering, University of Ljubljana, Tržaška 25, Ljubljana 1000, Slovenia

underlining the heuristic approach, but only in tackling the modelling ambiguity, which makes it different to other heuristic approaches, in general. In the pseudo Banach space, developed with the assumption that the process is fully measurable, the Lebesgue 2 normed space of continuous trajectories can be constructed. By the time exclusion, this space gives an ability to form the steady-state's subspace examined in [16]. It presents a unique approach to the prediction of control equilibrium, later on used in the boosting of the control dynamics of a simple and standard control solution. As such, the steady-state duty cycle based prediction is more accurate than the analytically driven for a wide range of operating points (OPs). Subsequently, in order to evolve the examination of the quantitative system's property to the quantitative/qualitative, this study presents a unique modelling principle of a DC-DC boost converter. This leads to a new control methodology called fuzzy model-based predictive control (FMPC), but with a minimized extra online processing complexity. In contrast to [16] the controller has better controllability in transients and completely excludes the set-point overshoots, rather it asymptotically approaches to the steady state, typically for the first-order system response.

The main controller's novelty is found in the combination of the Takagi–Sugeno (TS) fuzzy identification and the model predictive control (MPC). In contrast to any other known MPC approaches in the control of DC-DC boost converters, especially the most developed explicit model predictive control (EMPC) [6], that brings novelty in tackling the optimization problem strictly offline, this study builds a continuous model approximation of a hybrid dynamical system based on offline identification. Here, the advantage of the new methodology is twofold. First, it reduces the complexity by preserving the system order of the averaged-switch models, usually lifted by ambiguity variables [3,4,17], and at the same time reduces the number of regions in a robust sense. Second, it achieves better model accuracy for robust and especially physical cases where the switching period is equal to the sampling time. Finally, this method conciliates the grade of the accuracy, with either complexity or applicability. This is why that result encouraged the approach of identifying the system that has an already known, analytically driven and arbitrary accurate model for the robust solutions. The continuous system's approximation is afterwards presented in its discrete form as the base for the standard MPC problem with the preceding horizon principle. The state-space matrices are not analytically driven for the DC-DC boost converter, but these are time-dependent outputs of the fuzzy engine that heuristically correlates the previously identified regions. The experimentally rendered system's knowledge is presented by three arrays (27×5, 3×3 and 3×3) stored in the processor's memory. The simple online arithmetic extracts the knowledge written in the arrays of real numbers. All the convex optimization is made offline, and the online calculation's complexity is related to the typical MPC problems, but now with the matrices of a reduced rank of simpler linear model. The method is applicable for more complex multiple input multiple output (MIMO) systems, even in this study presented model is NARX MISO. The experiment with the MIMO identified model was found to be unnecessary, as a consequence of the models' accuracy comparison, which was either the approach in the selection of the regression vectors.

The MPC [18,19] in this study is a compact and standardized control solution for time-variable system matrices [20], and therefore advanced in the adaptive time-dependent cost function's suppression factor and represents suboptimal control with the avoidance of complex quadratic programming.

In contrast to some of the known fuzzy control solutions in PEC [9], using a simple fuzzy inference mechanism and ad hoc tuning, or advanced and complex fuzzy solutions in [10], this study underlies the heuristic approach in fuzzy modelling of Takagi–Sugeno [21] by reducing the number of rules in the rule base and allowing a deterministic formulation of the consequence functions, further used in the MPC. Some newer releases [22] are successful in employing a powerful fuzzy methodology, but this work is augmenting the paradigm in the modelling of the system's hybrid nature in order to minimize the online complexity and elevate the applicability.

As a conclusion, the experimental evaluation of the methodology gives the confirmation of aforementioned statements.

The paper is organized as follows. Section 2 starts with a paradigm in the modelling of the DC-DC boost converter and explains the basic problem in an analytical system examination. Section 3 explains the fuzzy model identification and points out the modelling features that are contrary to the established ones. Section 4 presents an overview of the applied control methods, which will be followed by the experimental results in Section 5. Lastly, Section 6 provides a short conclusion and a suggestion for future work on non-linear MPC in PEC by modelling of the system's hybrid nature in order to minimize the online complexity and elevate the applicability.

2. Paradigm in the modelling of a DC-DC boost converter

An example is taken from the literature [7,16], with all its numerical values, to be able to compare the results with previous research.

For the principal part of the electronic circuit in Figure 1, apart from the pulse-width modulator (PWM) and the controller with its set point s, by using Kirchhoff's voltage and current laws, we can form the ordinary differential equations (ODEs) $\dot{\boldsymbol{z}}(t) = f(\boldsymbol{z}(t)) + g(\boldsymbol{z}(t))u(t)$. By selecting the state vector $\boldsymbol{z}(t) = [v_C(t)\ i_L(t)]^T$ and the input as an independent voltage source $E(t)$, the mathematical model can be driven with the assumption that semiconductors are ideal switches and that the inductivity has no equivalent series resistance (ESR) [7]

$$\dot{\boldsymbol{z}}(t) = \boldsymbol{A_i z}(t) + \boldsymbol{B_i} E(t) \quad i \in [1,2,3]\ i - \textit{circuit topology}.$$

$$\boldsymbol{A}_1 = \frac{1}{C(R+r_c)}\begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix}, \boldsymbol{B}_1 = \boldsymbol{B}_2 = \begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix} \boldsymbol{B}_3 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{1}$$
$$\boldsymbol{A}_2 = \frac{1}{C(R+r_c)}\begin{bmatrix} -1 & R \\ \frac{-CR}{L} & \frac{-CRr_C}{L} \end{bmatrix}, \boldsymbol{A}_3 = \frac{1}{C(R+r_c)}\begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix}.$$

The mathematical modelling of switched-mode electrical circuits faces problems of discontinuity and all the related side effects or non-linearities [1].

Even if a simple switching algorithm is selected, the analytical definition of the duty cycle becomes a transcendental mathematical problem and it can only be solved by numerical methods. The reader is referred to the extensive literature [1,7–10], and references therein, that define the aforementioned problems.

Based on the authors' opinions and knowledge about this particular physical system, further well-established analytical modelling mostly develops in two different directions: first, already found in the earlier work of Midlebrook, Ćuk and Erikson [23,24], modelling by *small signal*

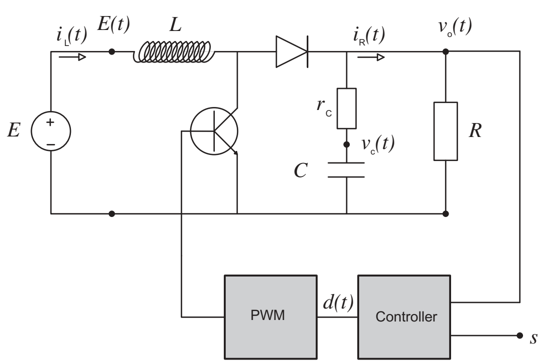

Figure 1. Typical voltage-controlled DC-DC boost converter.

models or *large signal models*, more from the side of elementary circuit theory; and second, also the one comprising the previously mentioned, but more analytically structured in the modern theory of modelling hybrid systems. The latter is conceptually connected to the system's piecewise linearity (PWL), with the implementation of logical variables, constraints, or inequalities as the product of the natural form of the system. A distinctive presentation on the equivalence of classes of hybrid dynamical models is available in [25]. It is still unknown regarding the methods that can be recognized as the most efficient and the general parameters that decide it.

Therefore, if we agree that the aforementioned assumptions (1) are negligible, we can proceed with the subsequent steps of modelling into the *hybrid system* model (2)

$$\dot{\boldsymbol{z}}(t) = \begin{cases} \boldsymbol{A}_1\boldsymbol{z}(t) + \boldsymbol{B}_1E(t) & kT_S \le t \le kT_S + t_{1,k} \\ \boldsymbol{A}_2\boldsymbol{z}(t) + \boldsymbol{B}_2E(t) & kT_S + t_{1,k} < t \le kT_S + t_{1,k} + t_{2,k} \\ \boldsymbol{A}_3\boldsymbol{z}(t) + \boldsymbol{B}_3E(t) & kT_S + t_{1,k} + t_{2,k} < t < (k+1)T_S \end{cases} \\ k = 0, 1 \ldots \infty. \tag{2}$$

Now, we simply recognize a *hybrid automaton* [2] with three discrete states defining a different continuous dynamics. In the hybrid system modelling, those discrete states are connected to the mode of the system's operation. From Equation (2), we find that the modes are defined by the converter's switching time period T_S and the subintervals $t_{1,k} + t_{2,k} + t_{3,k} = T_S$. The converter is driven by the PWM and the duty cycle $d = t_{1,k}{T_S}^{-1}$. Thus, our system is representative of the SAS. From the aforementioned, with a known T_S, a definition of the subintervals relies on the duty cycle, which is defined as a function of the state variables $\boldsymbol{z}(t)$. The problem that has to be handled by the hybrid system's formalisms is the prediction of the time subintervals. That is critical in the prediction of $t_{3,k}$ naturally controlled by the diode's (Figure 1) disconnection, which is a distinctive mode of the converter's functioning called DCM. The time of the transistor's on state is $t_{1,k}$. In the following sequence $t_{2,k}$ is the time of the transistor's off state, dependent on the natural diode's commutation that lasts $t_{3,k}$ before the next cycle T_S appears. Nevertheless, from the aspects of the control techniques the input signal to our model has to be the duty cycle, and in (2) the PWL continuous systems are based on the source voltage as an input. The well-established hybrid system formalisms take a DC-DC boost converter as an illustrative example [2], but from our point of view it is far from that.

After this section, and driven by the final goal to render the modern and intelligent control methodology for a non-linear dynamical system, we have to analyse the process and select a proper modelling formalism. More comprehensive and advanced control methodologies are based on a mathematical model of the process evolving to the model control solution. The compact and well-developed methodology is a MPC [18,19]. Our work is conducted in that direction. Here, the modelling discussion is also important.

The first concern emerging in the analysis of the modelling formalism (2) is that in our example of SAS the physical process is a combination of modes. The mode by itself has no physical background if it is not related to other modes that are in harmony with the physical meaning. If there is no exchange of hybrid modes, the electronic circuit does not function. To fulfil that, a controlled variable is a *limit cycle* [1] and it has to be observed from the point of view of its periodicity.

Second, the non-linearity of the system is not only its hybrid structure, but it is by assumption (1) also excluded from the non-linear circuit elements together with the anomalies caused by the energy dissipation. Our process is PEC and the main physical meaning is the energy transfer.

Third, a modelling problem is a multidisciplinary task and from the control aspects it should be driven by the final goal, the controllability and the stability of the process [20], which includes an exclusion of the non-linear phenomena [1].

Yield, a decomposition of the non-linear system problem to the PWL is plausible, but it has to be done carefully in order to avoid any unnecessary complexity and increase the accuracy in realistic applications.

As a conclusion about the objectives, we are focused on a system with a fixed switching period T_S equal to the sampling time, where $t_{1,k}, t_{2,k}, t_{3,k} < T_S$ are the times related to a different circuit topology. From the side of the non-linear dynamical system examinations, a general expression has to evolve in the yet unknown $\dot{\boldsymbol{z}}(t) = f(\boldsymbol{z}(t), d(t)) + g(\boldsymbol{z}(t), d(t))d(t)$, for the duty cycle $d = t_{1,k}{T_S}^{-1}$ as a control signal for closed-loop control and scalar input signal in the process. The duty cycle and PWM is the genuine part of our process and has to be modelled accordingly.

Subsequently, with the involvement of identification in the robust modelling of switched mode converters and the avoidance of strict assumptions, we propose refinements to the general modelling approach

$$\hat{\boldsymbol{z}}(k+1) = \begin{cases} \hat{\boldsymbol{A}}_1\hat{\boldsymbol{z}}(k) + \hat{\boldsymbol{B}}_1 u(k) \;\; if \;\; \delta_1(k) = 1 & \delta_i(k) \in \{0,1\} \quad \forall i = 1, \ldots, n_i \\ \vdots & \\ \hat{\boldsymbol{A}}_{n_i}\hat{\boldsymbol{z}}(k) + \hat{\boldsymbol{B}}_{n_i} u(k) \;\; if \;\; \delta_{n_i}(k) = 1 & \bigoplus_{i=1}^{n_i}[\delta_i(k) = 1] \;\; n_i - number\ of\ integer\ variables \end{cases} \tag{3}$$

$$\hat{\wp}_i \cap \hat{\wp}_j = \emptyset, \quad \forall i \neq j \quad \hat{\wp}_i - i^{th}\ polytop,$$

$$[\delta_i(k) = 1] \leftrightarrow \left[\begin{bmatrix} \hat{\boldsymbol{z}} \\ u \end{bmatrix} \in \hat{\wp}_i\right] \quad \bigcup_{i=1}^{n_i} \hat{\wp}_i = \hat{\wp}$$

which then evolves into general approximations of the non-linear dynamical system using the equation

$$\hat{\boldsymbol{z}}(k+1) = \sum_{i=1}^{n_i} \left[\hat{\boldsymbol{A}}_i \boldsymbol{z}(k) + \hat{\boldsymbol{B}}_i u(k)\right]\delta_i(k). \tag{4}$$

The reader is referred to [26] and the references therein.

The modelling approach (3), (4) is generally derived from (1) and (2) and finalized by the MLD modelling. To derive the final MLD expression for the DC-DC boost converter in Figure 1, a naturally hybrid system (2), or *hybrid automaton,* will be approximated by a *discrete hybrid automaton* [2]. The new formalism has to be seen as a final discrete-time model, as shown in (3) and (4) in the general sense. The MLD modelling is explained in detail in the literature throughout the HYSDEL framework [2,3,5,6,17]. The recognition of the integer variables $\delta_i(k) \in \{0,1\}$, and accordingly the appropriate model for $i = 0, \ldots n_i$, is based on the time that has to be shorter than the discrete time step, $kT_S < t < (k+1)T_S$. In the online processor's operation, this methodology assumes that the definition of $\delta_i(k)$ and the calculation of the $\hat{\boldsymbol{A}}_i$, $\hat{\boldsymbol{B}}_i$ matrices (4), that are the approximation of the system's operation in the contemporary linear region $\hat{\wp}_i \subset \hat{\wp}$, are possible during the time $t < T_S$. From the point of view of a fixed switching period T_S equal to the sampling time, the state-space matrices $\hat{\boldsymbol{A}}_i$, $\hat{\boldsymbol{B}}_i$ in (4) do not correspond to (1). That is not only because the matrices $\boldsymbol{A}_i$, $\boldsymbol{B}_i$ (1) are of the continuous space, it is also that $\hat{\boldsymbol{A}}_i$, $\hat{\boldsymbol{B}}_i$ in (4) are assumed to be discrete time $t = kT_S$ state-space matrices. Further on, those have to be accordingly predicted for the system's evolution through the time $kT_S < t < (k+1)T_S$ based on the predecessor control variable $u(k) = d(k)$. For a DC-DC boost converter, it is a result of *propositional logic* equations built on the multiple logic variables that are defined from the A/D and D/A variables' transformations during each time step kT_S. The general approach discussed elevates the original state-space model (1) by the involvement of the new discrete state variables, thus $\dim(\hat{\boldsymbol{z}}) > \dim(\boldsymbol{z})$. Hence, it necessarily affects the complexity of the state-space model and accordingly limits the applicability. Besides the underlined online complexity, the MLD model is valid for an assumption made in (1) added to ESR assumptions as a constructive part of the electronic circuit. Therefore, the modelling uncertainty that is characteristic for the example of this work is not completely grasped for natural processes.

The following work is carried out differently to find accurate modelling, which also preserves the robust and general knowledge of the system. Also, it results in a mathematical form that is subsequently applicable for the well-developed MPC. The method is based on a state measurable system, including the source and output current, transfers the main burden of non-linear dynamical system examination strictly to the offline problem with all its complexity. Hence, it can be simply considered as one of the EMPC methods [6].

The mathematical framework will not be exact and focused on the problem of *differential inclusion* and *complementarity formalism* [12,13,27], but rather on solutions in the pseudo norm vector space [16].

Theoretically, the idea is strongly supported in [28, Chapter 3], and elementarily connected to the approximate continuity and smoothing operation of the disjoint sets in the Lebesgue space.

If we now reconsider the averaging idea [23] to derive the local model, but numerically emphasizing the mathematical expression of the electronic circuit equivalent for the time $t \geq T_S$, we are smoothing the disjoint model structure in $t < T_S$. This smoothing operation, with the assumed measuring ability of $E(t)$, $i_L(t)$, $v_o(t)$ and $i_R(t)$, will, unlike the known analytical averaging process, find an approximation on a wider range of system parameter variation around the OPs. At the same time, the derived local model is a part of a new pseudo norm space $\wp$, and containing the discrete equivalents of approximately continuous functions $f_i(\boldsymbol{x_m}(k)) \subset \wp_i \subset \wp$. The edges of previous polytopes (3) are softened by fuzzy logic [16,21] and new-formed regions smoothly passing from one to another, tracking the system parameters' fluctuations.

Equation (3) now obtains a form different from any of the aforementioned analytically driven approaches

$$\begin{gathered}\boldsymbol{x_m}(k+1)=\sum_{i=1}^{p}[\boldsymbol{A}_{m_i}\boldsymbol{x}_m(k)+\boldsymbol{B}_{m_i}u(k)+\boldsymbol{R}_{m_i}w(k)]\beta_i(\boldsymbol{\varphi}_2(k))\\ \beta_i(\boldsymbol{\varphi}_2(k))\in[0,1]\quad \sum_{i=1}^{p}\beta_i(\boldsymbol{\varphi}_2(k))=1\quad i=1,\ldots,p,\end{gathered} \tag{5}$$

where the matrices $\boldsymbol{A}_{m_i}$, $\boldsymbol{B}_{m_i}$ are the new numerically identified state-space matrices, $\boldsymbol{R}_{m_i}$ is the residual matrix and $\beta_i(\boldsymbol{\varphi}_2(k))$ are the normalized degrees of fulfilment, which are explained in detail in Sections 3 and 4, together with reduced number of regions $p < n_i$. In the Equation (5) we can recognize the main difference with respect to a typical MLD approach based on (2) that is conceived in the normalized degree of fulfilment and new matrices. The former is a function of a regression vector $\boldsymbol{\varphi}_2(k)$, particularly common for an identification process and consisting of measured system variables in a time $t \leq kT_S$. In other words, the bivalent logic encoding of the uncertainties in (2) and (3) is evolved by the polyvalent *fuzzy logic*, more convenient for a complexity of realistic examples. Apart from that, the matrices $\boldsymbol{A}_{m_i}$, $\boldsymbol{B}_{m_i}$ and $\boldsymbol{R}_{m_i}$ are rendered by the identification process, performed without the subjective assumptions and simplification of an electronic circuit. The new vector of state variables $\boldsymbol{x_m}(k)$ is not augmented, but maintains a dimension of 2 as in (1).

However, the theory of *discrete hybrid automata* and in general the analytical modelling (3) of the physical system from Figure 1 was conducted using MATLAB [29] continuous/discrete functions in a hybrid simulation model [16]. Considering the processing power of a PC, and the advanced MATLAB tool with the possibility of *embedded functions*, the constructed model will provide an acceptably accurate approximation of the physical model and the base for the development of the subsequent methodology.

At this point, further examinations will be based on the numerical methods in the simulation and control design of the predictive control algorithm, and in contrast to most of the known developed methods, they do not consist of PWL expressions (1). By knowing that the major objective of the following work is the control of the output voltage, which is assumed to be a DC signal, the physical approach including the aforementioned leads us to the selection of the root-mean-square (RMS) measured values. Now, the new state variables cause a mathematical transformation of the original state-space (1) to the pseudo norm vector space. It is feasible and based on the assumption that a new state vector $\bar{\boldsymbol{z}} = [\bar{v}_o, \ \bar{i}_L] \in \mathbb{L}^2$ (Lebesgue), as a product of the numerical integration methods with an approximate solution in the system discontinuity [16]. In the continuation of this work the transformation will be made using MATLAB embedded functions applied to the simulation model. Accordingly, for a final experiment, we present a realistic counterpart of the numerical integration in the rendering of the Lebesgue 2 normed space. Figure 6 presents an integrated circuit (IC) AD637 that is built to provide continuous states of the transformed hybrid system state-space. Its role is to reduce the online processing workload, and to inherit the continuous system like the accuracy of the integration in the rendering of the RMS values. The RMS measurement of the original state-space variables and the measurement of $E(t)$ and $i_R(t)$ is carried out in the time period T_S, excited by PWM scaled duty cycle d_u.

3. Fuzzy identification of hybrid simulation model and converter

DC-DC converters are a good example of where the non-linear dynamical system appeals in the grasping of global system knowledge or modelling. The following work presents the global process modelling known as NARX (Non-linear autoregressive with eXogenous inputs) [30]. Guided by the modelling paradigm discussed in Section 2, the identification of the process analytically defined as the SAS (1), (2) will be performed on the hybrid simulation model [16] and the experimental process of a real DC-DC boost converter (Figure 6 and Table 1). Following the exceptional results [16] when modelling the complete space of stable OPs of a DC-DC boost converter, this work is conducted in the direction of finding a more superior methodology in identifying a qualitative pattern of an electronic circuit expressed in the non-linear dynamical model.

Underlying the previous sections, the decomposition of a complex non-linear problem will be made on the way to elevate the analytical model's accuracy for a physical case, where employing the identification methods diminishes the initial assumptions in the analytic approach. Similar to [16], and observing from the analytic aspects, the naturally two-dimensional space of the state variables $\boldsymbol{z}(t) = [v_C(t) \ i_L(t)]^T$ will be transformed into the multidimensional space of the Lebesgue 2 normed variables $\boldsymbol{V} = \{\bar{E}(t), \bar{i}_L(t), \bar{i}_R(t), \bar{v}_o(t), \bar{d}_u(t)\} \in \mathbb{R}^{5+1}$ using the measuring process. Now, our modelling will be formulated referring the objective knowledge collected in the process of identification. The pseudo norm applied to the measuring quantities $(\boldsymbol{V}, \ \| \cdot \|)$ in the simulation or simply the measurement of the RMS values in the physical process, in the fixed time intervals kT_S for $0 \leq k < \infty$, is transferring the complete problem from the original hybrid system (2) to the non-linear continuous system. Referring to the law of the conservation of energy, while we are

Table 1. Constructive elements of the examined and controlled DC-DC boost converter Figure 6.

Element	Code or value
Inductor L	211 μH
Capacitor C	222 μF
Transistor	FDB2532
Diode	RURP3020
Current shunt monitor	INA193
True RMS IC	AD637
Microcontroller	TMS320F28069

watching from the point of view of the fixed time interval it makes the energy a continuous function of time. Our DC-DC boost converter is PEC and the future controller's task is to transfer the energy to the consumer and preserve the fixed output voltage $\bar{v}_o(t)$. In a mathematical sense from the qualitative theory of a dynamical system the controller is controlling an output limit cycle $v_o(t)$ in the time period T_s [1]. Our identification task is to reveal the MISO periodic map that transfers the arbitrary number of measured states to the continuous variable $\bar{v}_o(t)$. In the transformed vector space $(\boldsymbol{V}, \ \|\cdot\|)$, a duty cycle $\bar{d}_u(t)$ is given as the continuous variable just in the general sense. Herein it is discrete, and its precision is as high as the controller's digit resolution. The future discrete controller's sampling time T_s genuinely transforms the new non-linear continuous system into a discrete time system.

The yet unknown non-linear system in its discrete form $\boldsymbol{x}(k+1) = f(\boldsymbol{x}(k), u(k)),\ y(k) = g(\boldsymbol{x}(k))$ is a complex mapping over the vector of transformed states $\boldsymbol{x} \in \mathbb{R}^2$ and the input $u = d_u \in \mathbb{R}$ into the output $y \in \mathbb{R}$, which will be identified as a T-S fuzzy model

$$y_m(k+1) = \boldsymbol{F}(\boldsymbol{\varphi}_k). \tag{6}$$

The Takagi–Sugeno fuzzy model, as a global model, approximates the non-linear function $y \in \mathbb{R}$ as a mapping $\boldsymbol{F}$ of the unknown regression vector $\boldsymbol{\varphi}_k$ in a time kT_s to the step ahead predicted output $y_m(k + 1)$. All this is possible by assuming that f, g are mapping over smooth functions consisting of a vector of states $\boldsymbol{x} \in \mathbb{R}^2$ in a space $\mathbb{R}^l$. Concerning our DC-DC converter as a process, the state variables are now RMS-transformed continuous functions $\boldsymbol{x} \in \mathbb{L}^2 \subset \mathbb{R}^l$.

The identification of non-linear systems in this work is a continuation of previous studies [31–33] based on a heuristic approach by implementing the fuzzy identification as a *Universal Approximation* [34,35]. Equation (6) is reconstructed throughout the two grades of the identification, and hence,

$$y_m(k+1) = \boldsymbol{\beta}(\boldsymbol{\varphi}_2(k))\boldsymbol{\theta}_1\boldsymbol{\varphi}_1(k)^T. \tag{7}$$

Physical knowledge of the system that groups this identification process in the *grey box* identification [30], helps us in the selection of the regression vectors and those typically consist of measured values in a time $t \leq kT_S$. The regression vector components are variables of the vector space $(\boldsymbol{V}, \ \|\cdot\|)$ and for a matter of simplicity in the sequel we avoid a line over the symbol. A several experimentally compared selections of regression vectors proposed $\boldsymbol{\varphi}_1(k) = [v_o(k)\ v_o(k-1)\ i_L(k)\ d_u(k)\ 1]$ and for $i_R(k) > 0$ $\boldsymbol{\varphi}_2(k) = [E(k)\ v_o(k)/i_R(k)\ i_L(k)]$. A comparison is made bearing in mind the model/system error, and the indices 1 and 2 correspond to different grades of identification. The systematic approach was not built at this point as the problem of identification in PEC may differ greatly from the process to the process with distinctive physical characteristics. A prior knowledge of the physical system in general reduces the task extents. Two selected regression vectors are members of a developed *fuzzy logic identification* structure. However, these can be separately examined, as the $\boldsymbol{\varphi}_1(k)$ has more a local impact and the $\boldsymbol{\varphi}_2(k)$ a global impact. For example, that circumstance is designed and used in our approach.

Therefore, $\boldsymbol{\theta}_1$ denotes the parameter matrix of the first-grade identification, resulting in a set of p-identified ARX linear models (5), evolved from the number of rules in the fuzzy rule base and equal to the number of OPs. For example, ${}_1\boldsymbol{\theta}_{11}$ is the vector of the model coefficients of the first operating range. Despite the ability to use an averaged mathematical model (1), (3) in certain OP of the DC-DC converter, which would be derived by the perturbation method at that point, we propose a linear quadratic problem and a least-squares identification method around the selected point.

This approach gives a predicted and better variance of the model error, especially by observing from applicability aspects on physical models. Thus, linear models around the OPs will be identified by the least-squares method

$$ {}_{1}\boldsymbol{\theta}_{n,1} = (\boldsymbol{\Psi}_n{}^T\boldsymbol{\Psi}_n)^{-1}\boldsymbol{\Psi}_n{}^T\mathbf{Y}_n, \tag{8} $$

where $\boldsymbol{\Psi}_n \in \mathbb{R}^{S\,X5}$ is a matrix of measured S training samples of the regression vector $\boldsymbol{\varphi}_1$ components for the operating point n and $\mathbf{Y}_n \in \mathbb{R}^S$ is the vector of the step ahead responses (MISO processes). To gain the OP training data set (8), the physical model is primarily tested to experimentally define the steady-state $d_{u,n}$ for the n[th] OP. This gives us the centre duty cycle, which is expanded in the excitation function $d_{u,n}(t)$ for the n[th] OP region that has to be identified.

The process of fuzzification is carried out based on *Gaussian membership* functions followed by a *product* to represent the conjunction in the premise and ending with the typical *centre-average* defuzzification [36]. If we select $\boldsymbol{\mu}_{\mathbb{U}1}$, generally, the matrix of the degree of fulfilment, as a vector $\boldsymbol{\mu}_{\mathbb{U}_1} = \left[e^{-\frac{1}{2}\left(\frac{\varphi_{2,1}-c_{1,1}}{\sigma_{1,1}}\right)^2} \; e^{-\frac{1}{2}\left(\frac{\varphi_{2,1}-c_{1,2}}{\sigma_{1,2}}\right)^2} \; e^{-\frac{1}{2}\left(\frac{\varphi_{2,1}-c_{1,3}}{\sigma_{1,3}}\right)^2} \right]$, $\boldsymbol{\mu}_{\mathbb{U}i} \in \mathbb{R}^3$ also means that in our example, the linguistic variable 'source' has three Gaussian membership functions. After the selection of the same for the other two inputs of $\boldsymbol{\varphi}_2(k)$, the fuzzy rules

$\mathcal{R}^{\boldsymbol{i}}$ *for* $i = 1...p$, $p = \dim(\boldsymbol{\mu}_{\mathbb{U}_1}) \cdot \dim(\boldsymbol{\mu}_{\mathbb{U}_2}) \cdot \dim(\boldsymbol{\mu}_{\mathbb{U}_3}) = 27$, and form the rule base (9). The set of membership functions $\{\mathbb{U}_{1,1}, \mathbb{U}_{1,2}, \mathbb{U}_{1,3}\} \subset \mathbb{U}_1$ is a subset of the input universe of the discourse for the linguistic variable 'source' $E(k)$, and, similarly, $\mathbb{U}_2$ and $\mathbb{U}_3$ are universes of discourse for the 'load' $v_o(k)/i_R(k)$ and the 'coil current' $i_L(k)$, respectively.

As a product of the selected fuzzy construction, the vector of normalized degrees of fulfilment in fuzzy mapping (7) is presented by (10) in our example for *j = 1,2,3* (fuzzy inputs). The vector $\boldsymbol{\beta}(k) \in \mathbb{R}^p$ has a length equal to the number of rules in the rule base of the fuzzy model, and the symbol $\otimes$ is the Kronecker product.

$$ \begin{aligned} &\mathcal{R}^{\boldsymbol{i}} : IF \quad E(k)\, is\, \mathbb{U}_{1,j_1} \;\; AND \quad v_o(k)/i_R(k) \;\, is \;\, \mathbb{U}_{2,j_2} \;\; AND \quad i_L(k) \;\, is \;\, \mathbb{U}_{3,j_3} \\ &THEN \quad y_m(k+1) = a_{i,1}v_o(k) + a_{i,2}v_o(k-1) + a_{i,3}i_L(k) + a_{i,4}d_u(k) + a_{i,5} \\ &for\ i = 1,\dots,p \quad and\, unique\, set\, [j_1, j_2, j_3]^i \;\; if \;\; j_1, j_2, j_3 \in \{1,2,3\} \end{aligned} \tag{9} $$

$$ \boldsymbol{\beta}(\boldsymbol{\varphi}_2(k)) = \frac{\left[\boldsymbol{\mu}_{\mathbb{U}_1}(\varphi_{2,1}(k)) \otimes \boldsymbol{\mu}_{\mathbb{U}_2}(\varphi_{2,2}(k)) \otimes \cdots \otimes \boldsymbol{\mu}_{\mathbb{U}_j}(\varphi_{2,j}(k))\right]}{\left\|\boldsymbol{\mu}_{\mathbb{U}_1}(\varphi_{2,1}(k)) \otimes \boldsymbol{\mu}_{\mathbb{U}_2}(\varphi_{2,2}(k)) \otimes \cdots \otimes \boldsymbol{\mu}_{\mathbb{U}_j}(\varphi_{2,j}(k))\right\|_1}. \tag{10} $$

Altogether, in the second grade of identification, we constructed $\boldsymbol{\theta}_2 = \{\boldsymbol{\theta}_1,\ \boldsymbol{c}, \boldsymbol{\sigma}\}$ as a set of fuzzy model parameters for $\boldsymbol{c}, \boldsymbol{\sigma} \in \mathbb{R}^{3x3}$, where $\boldsymbol{c}$ (centres) and $\boldsymbol{\sigma}$ (standard deviations) denote the matrices of the membership function parameters. The vector $\boldsymbol{\beta}$ is a vector of normalized values (10), and thus

$$ \sum_{i=1}^{p} \beta_i = 1. $$

To optimize the selected T-S fuzzy model or to precisely define the parameters in the second grade of identification, the *gradient tuning method* was applied. By minimizing the cost function $J = \sum_{i=1}^{M} \frac{1}{2}(y_m(\boldsymbol{\varphi}_2, \boldsymbol{\varphi}_1|\boldsymbol{\theta}_2)^i - y^i)^2$ over the training set of data $\{(\boldsymbol{\varphi}_2, \boldsymbol{\varphi}_1)^M, y^M\} \in \boldsymbol{\Gamma}$, the overall parameters of the fuzzy model $\boldsymbol{\theta}_2$ could be tuned [36]. The simulation process resulting in the definition of the training data set $\boldsymbol{\Gamma}$ can be performed after the selection of the excitation input functions of the duty cycle $d_{u,b}(t)$, the source voltage $E_b(t)$ and the resistance $R_b(t)$. First, the *b = 1* set of simulation input functions is used for the training of the final construction of the fuzzy model and other sets, mostly for the evaluation process.

The parameters defined in the first grade of identification $\boldsymbol{\theta}_1$ are not expected to differ to a large extent by performing the $\min\limits_{\boldsymbol{\theta}_2}(J)$ convex optimization in the second grade of identification.

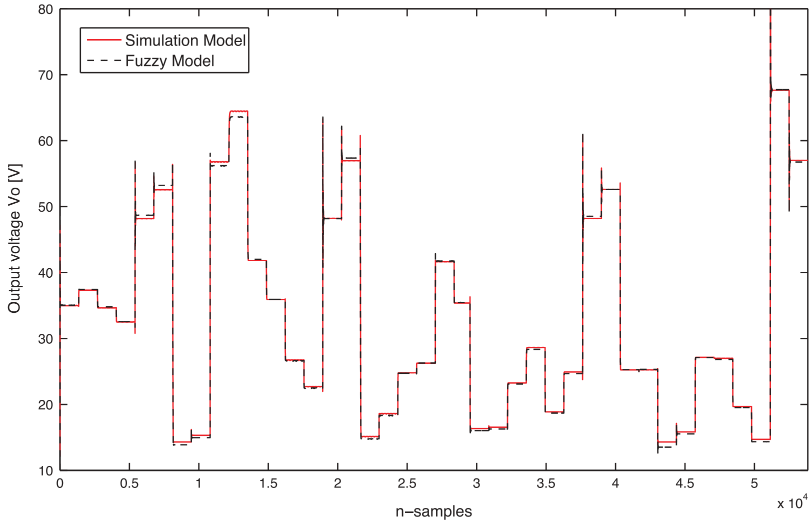

Figure 2. Results of comparison simulation combining v_o and y_m (the identified simulation model) for the 53843 testing samples; $SSE = 5.93 \cdot 10^3$ $MSE = 0.1101$.

To a much larger extent, we expect differing of heuristically chosen initial values of $\boldsymbol{c}$ and $\boldsymbol{\sigma}$. Accordingly, to accomplish the final fuzzy model by faster convergence, the step sizes related to the membership function parameters ($\boldsymbol{c}, \boldsymbol{\sigma}$) are bigger.

In a non-linear dynamical system identification, the minimum of the convex programming is, in some of the OPs, just a rough approximation. Our approach with two grades of identification, the least-squares and gradient method of identification, gives special attention to the selection of the identification sets of the data in both grades. The validation test results are presented in Figures 2 and 3.

The final equation of the fixed Fuzzy NARX (FNARX) model gained by the offline identification for the process of the DC-DC converter is

$$\begin{aligned} y_m(k+1) &= a_{m1k}v_o(k) + a_{m2_k}v_o(k-1) + a_{m3_k}i_L(k) + a_{m4_k}d_u(k) + a_{m5_k} \\ y_m(k) &= v_o(k) + e(k) \quad e - prediction\ error, \end{aligned} \tag{11}$$

where $\boldsymbol{a}_m(k) = [a_{m1}(k)\ a_{m2}(k)\ a_{m3}(k)\ a_{m4}(k)\ a_{m5}(k)]$ denotes the vector of the model's time-dependent coefficients. These are defined for each step of the control $\boldsymbol{a}_m(k) = \boldsymbol{\beta}(\boldsymbol{\varphi}_2(k))\boldsymbol{\theta}_1$ that takes care of the model adaptive tracking of the process' dynamical changes.

3.1 *Evaluation of new modelling vs. the established methods*

Except for the model evaluation carried out by typical identification framework, it is not less important to point out the main novelty featured in the new method vs. the already well-established methods.

The presented 'troublesome' identification is primarily bringing more precise modelling in processes with lower processing capabilities, where the sampling time is equal to the switching period T_S, and conciliating the control efficiency with its complexity. Most of the present methods are originally based on averaging and building the models on the typical involvement of an integer variable $\delta_i(k) \in \{0, 1\}$ by assuming that the two-circuit topologies exchange happened in an instant time $t \equiv 0$, but in reality $t \le \varepsilon$ for $\varepsilon > 0$. The topologies in that time are physically correlated, which brings the necessary complexity in analytical examinations. For a discussion on this topic the reader is referred to [12,13,27] and the references therein.

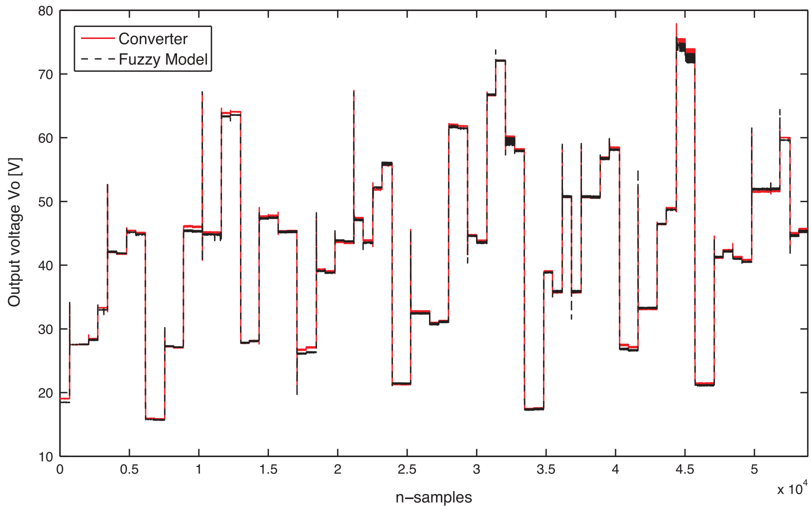

Figure 3. Results of a comparison of the experimental prediction y_m (the identified converter) and v_o as the process output on the 53843 testing samples for the real physical system Figure 6; $SSE = 8.53 \cdot 10^3$ $MSE = 0.1584$.

Presenting the system by typical linear or bilinear state-space presentation is just a continuation of the well-known averaging method [37]. Accordingly, to make a meaningful comparison, three differently built models, that is, the hybrid simulation model (2), the identified fuzzy nonlinear dynamical model (5) (in this work) and the analytically linearized averaged-switched model of the DC-DC boost converter are simulated and excited by the sinusoidal function around the OP. Based on the MATLAB [29] ID toolbox, the model responses' data gave the expected results. The drift effect related to the gain and the phase margin of the averaged-switched model is obvious on the sinusoidal excitation in the duty cycle range of $d_{u,OP}$ ±0.004, while the fuzzy identified model tracks the original hybrid simulation model with estimated preciseness, as shown in Figure 4. Furthermore, this comparison is carried out in CCM OP, where the standard analytical approaches [37] based on one integer variable (integer programming) have a comparable preciseness. The superiority is more obvious in DCM, where the identified model still preserves the same accuracy. Again with assumption that other methods are also based on the fixed switching period equal to the sampling time.

4. Applied control methods' overview

The fuzzy identification of the DC-DC boost converter derived finite impulse response model (FIRM) is generally in the form (11) or also typically called input/output model [19], where the indices k denote a time-variable linear model.

By assuming a full state measurement system, a novel approach in the research of the DC-DC boost converter is the fact that the current of a primary circuit $i_L(k)$ will be obtained by a further mathematical modelling assumed to be a part of the measured disturbance matrix $\boldsymbol{R}_{m_k}$. The $\boldsymbol{R}_{m_k}$ also consists of the *residual component* a_{m5_k} of our identified model (11). The following control algorithms based on a fuzzy internal model are derived for only one natural state variable that is simultaneously the output and controlled variable.

The MPC problem will be solved online in one scan of the processor time k for the model (5), which is now an approximated linear model (11) in the state-space form $y_{\boldsymbol{m}} = \boldsymbol{C}_{m_k}\boldsymbol{x}_m$. The state-space matrices at time $t = kT_S$ are $\boldsymbol{A}_{m_k} = [a_{m1_k}, a_{m2_k}; 1, 0]$, $\boldsymbol{B}_{m_k} = [a_{m4_k}; 0]$, $\boldsymbol{R}_{m_k} = [a_{m3_k} i_L(k) + a_{m5_k}; 0]$ and $\boldsymbol{C}_{m_k} = [1, 0]$.

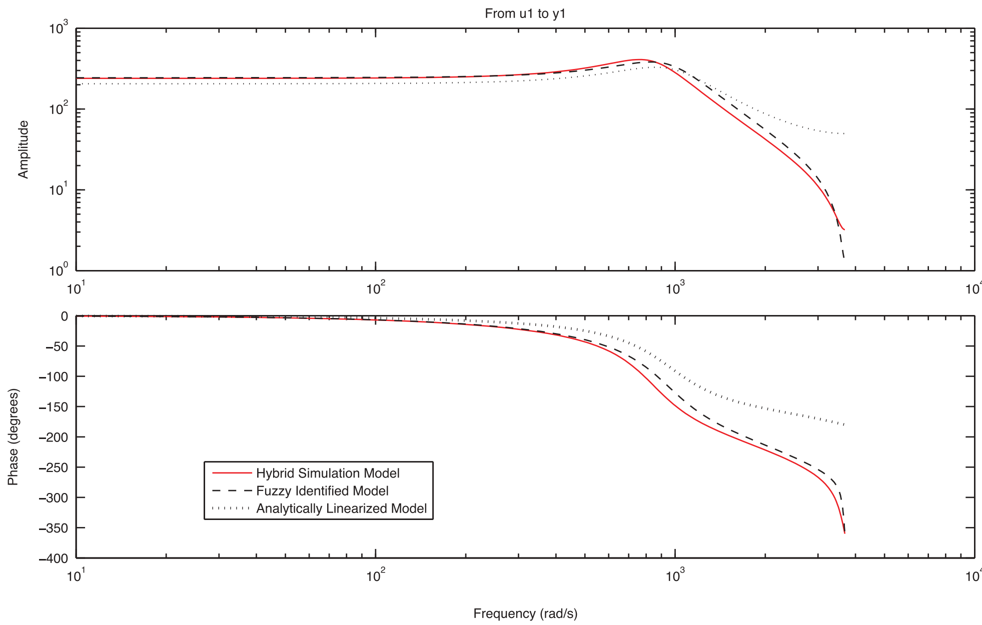

Figure 4. Comparison of second-order identified linear models in narrow regions around the OP, identification vs. analytical averaging.

Therefore, the vector of state variables is $\boldsymbol{x}_m(k) = [v_o(k); v_o(k-1)]$, input $u(k) = d_u(k)$ and the unit step disturbance $w(k) = 1$. This will be a basic representation of the internal model in the following *Finite Horizon* FMPC algorithms applied in the simulation and experiment.

The MPC methods are traditionally considered for the processes with the slow dynamics. Their limitation can be recognized in the computing complexity, which is a multilayer problem. Despite being aware of this limitation, our work was driven by the final goal that is implementing a MPC into the processes with faster dynamics. In the last 20 years the MPC methodology has been developed to become the most dominant in process technology. It was given great attention by academia. That developing process has constructed a control framework as a result of long-term researches in the field of control techniques. Therefore, it is a respective control solution for a broad number of different processes. Unfortunately, it is too pragmatic to expect that one control toolbox can grasp all possible natural processes and automatically devise the most efficient way of controlling them. We refer the reader to [6] and the references therein, where the authors present a survey of EMPC. That work certainly gives the diversities of the MPC complexity, in general.

Shown in [16] and continued in this work, the physical system's extents and constraints do not necessarily lead to the complexity of the applied control technique. In our work we found the MPC method more as a methodology than a single technique [19]. We have to underline its compact and standardized framework in looking for a stable control technique in the robust sense. The main drawback of the method is its complexity. As discussed in [6] the EMPC emerges as the solution to that problem. It suggests a transferring of the online processor's work to the offline regime. Our methodology is correlated to that idea. The optimization algorithms or solvers are the biggest processing time consumer. Because of that fact we performed a different recognition of the problem for an example of this work. Our online processing related to the MPC method is reduced to the *unconstrained standard predictive control problem* [18,19]. It is driven to the complete exclusion of the optimization algorithms and it can be considered as the suboptimal control. The MPC online optimization is derived by the basic linear algebra problem $\partial J(k)/\partial u(k) = 0$, where $J(k)$ denotes the performance index and $u(k)$ an input variable. A DC-

DC boost converter's input variable $u(k) = d_u(k)$ is constrained with the interval $[0, \max(d_u(k)) < 1)$. This critical constraint should not be a problem of the control algorithm, but rather a problem of the physical process, and it is tackled by the process itself. Although an unrealistic combination of the process parameters can lead to a violation of that constraint, the fuzzy identified model will even in that case saturate the control variable to the assigned margins. Furthermore, the state variables' constraints are grasped by the suppression factors of the MPC performance index, to decelerate a quick controller's response. In the case of the high peaks of the state variables, which are the characteristic events of the PEC, our controller will be disturbed proportionally to the energy level of the particular transient. This is obvious by recalling the external RMS measured values. The unexpected peaks are products of the process parameters' change and it is sufficiently treated by the method's *reference model* interpolation. The complex LMI is considered unnecessary, if the previous modelling work has achieved the most accurate linear approximation of the system at the particular OP. In the absence of any processing time 'luxury' that assumption performs acceptably. The time of the processing is not only burdened by the optimization as a cumbersome solution, but also by the rank of the process model. Our suggested modelling solution preserves the original rank of the system, characteristically for the traditional state-space averaging. The global model identification strategy discussed, and based on the objective physical constraints render the explicit control solution in the sense of the EMPC main goal. In contrast, the physical constraints are not simply assumed as being fixed. The online T-S fuzzy logic will select the consequence linear function that is a product of a measurement at the time kT_S, and treat a realistic violation of the assumed constraints. Using a case-oriented implementation of the MPC, we reduce the processing complexity, but do not endanger the objectivity and the accuracy. Reducing the problem of the non-linear dynamical system to the PWL constrains the MPC on the short horizon prediction and control solution. So, a physical limitation should work in the sense of the MPC reduction complexity in combination with the aforementioned.

4.1 *Fuzzy dynamic matrix control*

The first and the most basic MPC is the one strictly based on the dynamic Matrix [18,32,38]. As our system is proven to be open-loop stable, the DMC can be performed over FIRM (11) by transformation in the finite step response model (FSRM), which is one of the main characteristics of the DMC algorithm. The control law has to minimize the performance index

$$J(k) = \sum_{i=1}^{N_u} q_i(y_m(k+i|k) - r(k+i|k))^2 + \sum_{i=0}^{N_u-1} \lambda_{i+1}(k)\Delta u(k+i|k)^2. \tag{12}$$

4.2 *Fuzzy predictive functional control*

The predictive functional control (PFC) method originally developed by Richalet [39,40], and further in [41], has been chosen exactly because of its main distinction, when compared with the already explained DMC method. Thus, the reduction of the algorithm calculation workload and the simultaneous achievements in fast response processes make this method very suitable for the objectives given in the example of this work. The control law has to minimize the performance index

$$J(\hat{\boldsymbol{u}}, k) = \sum_{j=1}^{n_H} \left(y_R(k+H_j|k) - y_m(k+H_j|k)\right)^2 + \sum_{j=1}^{n_H} \lambda_j{}^2(k)\hat{\boldsymbol{u}}(k+j-1|k)^T\hat{\boldsymbol{u}}(k+j-1|k). \tag{13}$$

Both selected finite-horizon MPC methods and their *objective functions* or *performance indexes* consist of the suppression factor λ applied to the manipulated variable. Furthermore, it is time dependent and updated for each scan time by the controller's algorithm. The update or $\lambda(k)$ is derived from the simple proportional dependence of the predicted process gain in the time $(k+1)$ T_s, and hence, $\lambda(k) = a_{SUPP} \cdot y_m(k+1) \cdot u(k)^{-1}$, tuned by a_{SUPP} parameter.

The involvement of the suppression factor in the MPC methods is mostly used to achieve smoother control [38,40].

5. Simulation and experimental results

The identified fuzzy model based on an examination of the simulation model is integrated into the control algorithms and examined on the simulation model of Figure 5. So, fruitful results rendered by this examination led us in an experimental evolution of the identification and control developed methods on the physical system Figure 6. The selection of the electrical components Table 1 (T_s = 333 μs) is conducted in a sense to achieve a meaningful comparison with the similar expertise from the literature [7,16], as generally the complete article.

The new and experimental identification training data set $\{(\boldsymbol{\varphi}_2, \boldsymbol{\varphi}_1)^M, y^M\} \in \boldsymbol{\Gamma}$ is constructed by using the same microcontroller and its storage place Figure 6 in an open-loop regime. A process of convex programming in the minimization of the cost function $\min_{\boldsymbol{\theta}_2}(J)$ is programmed and executed on the standard laptop. As expected, the identification results are less accurate than those based on an examination of the simulation model, but also very much applicable for the final construction of the control algorithm. Figure 3 presents the comparison of the measured testing data set and the '*1 step ahead prediction*' based on the experimentally identified fuzzy model. As the new methodology is seeking for a robust solution, so the process of identification is more appealing in a sense of the test-bench preparation. The major part of the identification process of such a robust system is laying in the construction of the variable sources capable of sustaining substantial surges in the current and instabilities caused by that. This problem is certainly influencing the final model accuracy, but on the other hand fortifying the methodology and its applicability in realistic systems.

Because of the promising simulation results and a positive experience with the developed Fuzzy Model Predictive Controls, only FPFC will be presented. FDMC and FPFC, which were showing similar results in the simulation of control, but simultaneously the FDMC is more tedious in terms of calculations. The implementation of FDMC, because of the method's construction, limits its applicability on more powerful microcontrollers. The main limitation is strictly connected with

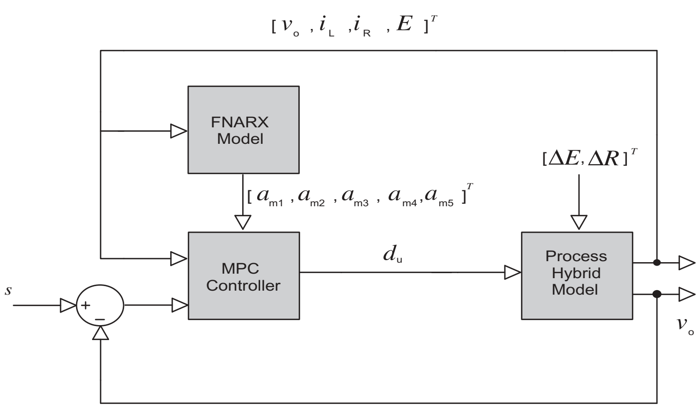

Figure 5. Simulation model for FMPC applied methods.

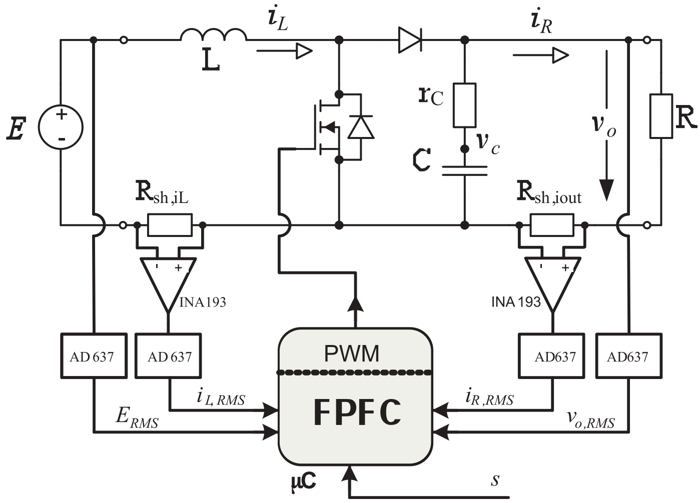

Figure 6. Experimental process for FMPC applied methods.

the prediction and the control horizon, which builds up complexity in the calculation of the inverse matrix and matrices' multiplications. One would say that the main processing time of this method would be the accessibility to the offline-learnt knowledge, but this experimental example occupies just 1.6% of the sampling time calculated for the worse-case scenario, if the processor for each data word uses the accessing time to the internal memory (which is Paged Flash access time). For the used processor that would be 5.5 μs. Also, the complete system's learnt knowledge is compressed into 153 data words in the processor's internal memory of 4G words. Even though the tedious work of the identification is conducted offline, which is related to the static part (6), the comprehensive and the accurate model of the contemporary process OP has to be selected online, as it is a dynamic function of regression vectors (7). The FNARX model parameters' vector $\boldsymbol{a}_m(k)$ in Figure 5 is calculated throughout the processor's *fuzzy engine* from the offline rendered global and distinctive knowledge. That significant processor's workload together with the FPFC devised optimal control signal takes approximately 180 μs of the processor's time of execution.

The PI controller, purely based on MATLAB tools for SISO controllers [29], was back-compared with the new derived FPFC, without any loss of generality and presenting the overall reference to any known, modern control solution. It was gained by the auto-tuning method, based on a singular frequency and minimization of the Integral Time Absolute Error (ITAE), and subsequently optimized by the gradient descent algorithm for a medium-scale performance. A meaningful comparison was derived by the process step parameter changes, for the wider operating range of the DC-DC converter. By applying the variable set point *s*, the converter will be guided from the DCM operation to a CCM, where the highest process gain is expected.

The disturbances of the process parameters are commenced simulating the possibly realistic DC-DC converter's operating regime.

The internal model discrepancy grows with the prediction horizon, but it is strongly influenced by the construction of the fuzzy model, where the inductor current is the measured value and assumed to be constant during the prediction horizon. This is also the reason why the prediction and control horizon are limited to the low value of $N_c = N_u = 12$. The stability is improved with the selection of the objective functions and fine-tuning of the manipulated value suppression factor. This is, for example, atypical in PFC. Furthermore, the tuning of the suppression factor and the construction of its time-dependent function preserves the suboptimal solution.

The offline optimized PI controller is comparable in the process with a higher gain range, where the optimization is carried out. Analogically, it is incomparably slow in the lower gain range. Figure 7 and a detailed Figure 8, from the oscilloscope, present the responses together to simplify the comparison. The FPFC method shows its robust advantage and for a wide range of different OPs performs similarly in its aggressiveness and steady-state stability. The main difficulty can be found in the steady-state error, which was not obvious in the simulation results where the model/process error was negligible, but also in the online processing demands that are incomparable higher than for the PI. In realistic extents the error reflects in a significant steady-state offset. It is also a feature of the developed methodology to tackle this kind of problem and compensate for it. Model-based control gives us the ability to use the predecessor model/process error data in forming a simple correction to the reference model.

5.1 *Fuzzy predictive functional control parameters*

The *step* and *parabola* selected basis functions are the most suitable in accordance with the process response on step changes of manipulated variable. These functions request two coincident points $n_H = 2$, H_1 and H_2 to be able to construct a feasible MPC problem, and hence, $H_1 = 1$ and $H_2 = 12$.

The reference trajectory y_R coefficient is $a_r = 0.01$ and the suppression factor $\lambda(k) \in [40,400]$.

5.2 *PI offline optimized parameters*

The PI controller parameters are inherited from [16].

Discrete equivalents are achieved based on a theoretical sample time to be as close as possible to the continuous form ($t_{sample} = 10^{-6}\ s$)

$$G_{PI}(z) = 5.648 \cdot 10^{-4}(z - 1.000354)(z - 1)^{-1}. \tag{14}$$

The offline optimization is carried out around the OP $s = 50$ V, $E = 10$ V, $R = 12.5\ \Omega$, *for*

$$G_m(z) = (-0.090237z + 0.0904)\left(z^2 - 2z + 0.9996\right)^{-1}. \tag{15}$$

6. Conclusion

The EMPC certainly brings a standardized control toolbox that is applicable on PEC nonlinear dynamical systems. Different modelling may minimize the main drawback in the complexity. In this study, we emphasized the idea that fuzzy modelling, as a universal approximation, is an applicable method. The presented complexity of the identification and the time-consuming convex programming was integrated into a global model derived from offline algorithms, which allowed an expectedly shorter execution time, but still preserved the adaptive tracking of the process dynamic changes. For each time scan, the fuzzy model produced the closest approximated linear model in a form that is applicable for the employment of standard finite horizon MPC methods, which then performed similarly and achieved all the control objectives. The selection of the performance index, including the suppression factor, was acknowledged to have high importance in guarding the stability and robustness of the constructed control algorithms. To fortify the knowledge gathered by the simulation, the complete methodology was tested on a real process of a DC-DC boost converter. With the experimental results we built a firm basis for further investigation and

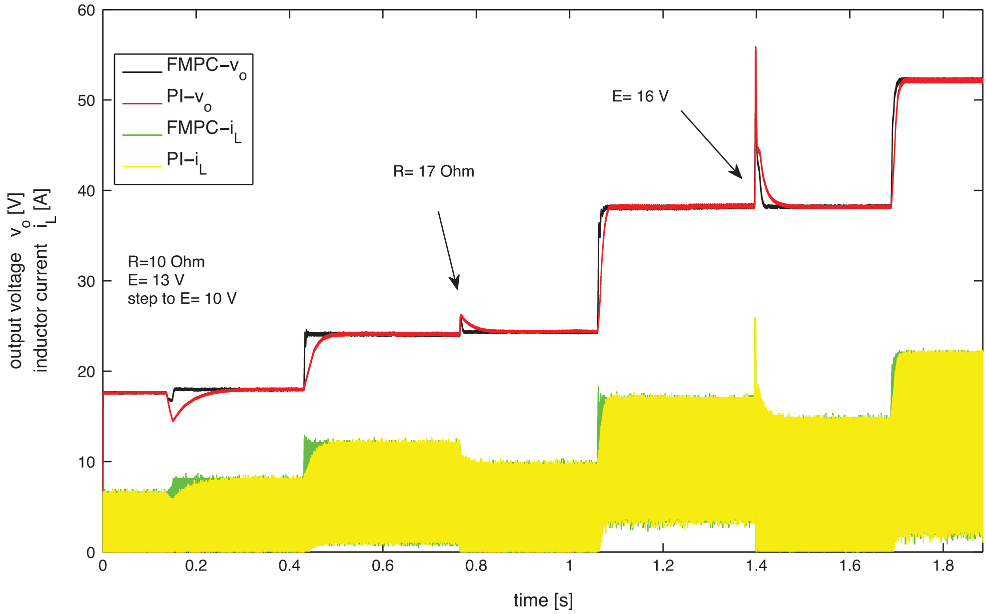

Figure 7. Experimental results of PI and FMPC controllers on step changes of the process parameters and reference point.

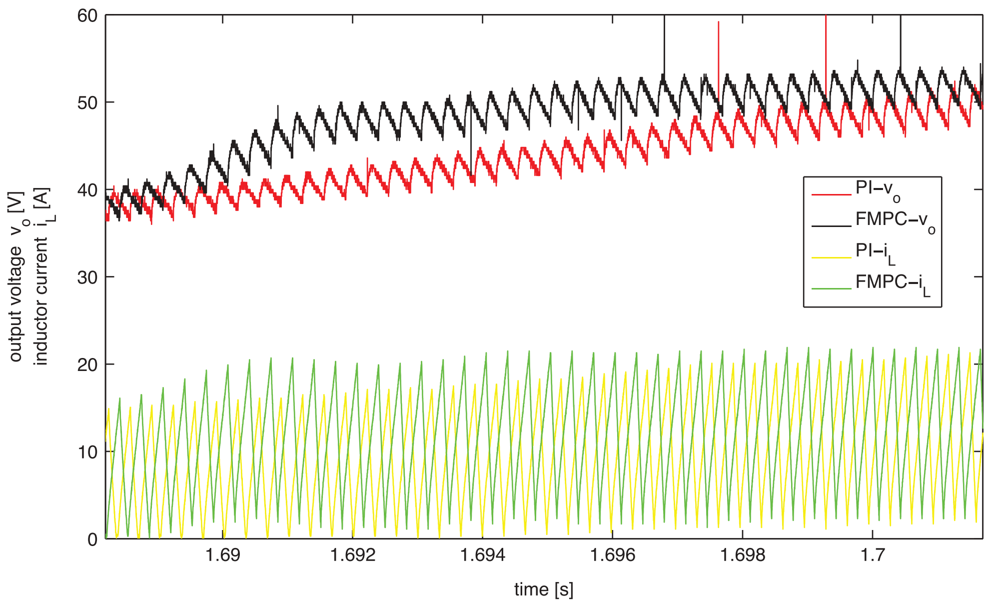

Figure 8. Detail from Figure 7, transient on the reference change.

method's wider applicability on more complex SAS. Further work should be carried out to achieve a meaningful discussion of the stability and comprehensive exclusion of nonlinearity problems.

Disclosure statement

No potential conflict of interest was reported by the authors.

References

[1] M. Di Bernardo, C.J. Budd, A.R. Champneys, and P. Kowalczyk, *Piecewise-Smooth Dynamical Systems – Theory and Applications*, Springer- Verlag London Limited, London, 2008.
[2] J. Lunze and F. Lamnabhi-Lagarrigue, *Handbook of Hybrid Systems Control; Theory, Tools, Applications*, Cambridge University Press, New York, 2009.
[3] S. Mariethoz, S. Almer, M. Baja, A.G. Beccuti, D. Patino, A. Wernrud, J. Buisson, H. Cormerais, T. Geyer, H. Fujioka, U.T. Jonsson, C.-Y. Kao, M. Morari, G. Papafotiou, A. Rantzer, and P. Riedinger, *Comparison of hybrid control techniques for buck and boost DC-DC converters*, IEEE Trans. Control Syst. Technol.. 18 (5) (September 2010), pp. 1126–1145. doi:10.1109/TCST.2009.2035306
[4] H. Fujioka, C.-Y. Kao, S. Almér, and U. Jönsson, *Robust tracking with performance for PWM systems*, Automatica 45 (2009), pp. 1808–1818. doi:10.1016/j.automatica.2009.03.026
[5] A.G. Beccuti, G. Papafotiou, R. Frasca, and M. Morari, *Explicit hybrid model predictive control of the DC-DC boost converter*, In: IEEE, PECS, Orlando, FL, 2007, pp. 2503–2509.
[6] A. Alessio and A. Bemporad, *A survey on explicit model predictive control in nonlinear model predictive control*, LNCIS. 384 (2009), pp. 345–369. Springer-Verlag, Berlin Heidelberg.
[7] C.K. Tse, *Flip bifurcations and chaos in three-state boost switching regulators*, IEEE Trans. Circuits Syst. 1 Fundam. Theory Appl. 41 (1) (January 1994), pp. 16–23.
[8] Y. Chen, C.K. Tse, S.-C. Wong, and -S.-S. Qiu, Interaction of fast-scale and slow-scale bifurcations in current-mode controlled DC/DC converters, Int. J. Bifurcat. Chaos 17 (5) (2007), pp. 1609–1622. doi:10.1142/S021812740701794X
[9] K. Guesmi, N. Essounbouli, A. Hamzaoui, J. Zaytoon, and N. Manamanni, *Shifting Nonlinear Phenomena in a DC-DC Converter Using Fuzzy Logic Controller*, Elsevier, Amsterdam, 2007.
[10] K. Mehran, D. Giaouris, and B. Zahawi, *Stability analysis and control of nonlinear phenomena in boost converters using model-based Takagi–Sugeno fuzzy approach*, IEEE Trans. Circuits Syst. I Regular Pap. 57 (1) (January 2010), pp. 200–212. doi:10.1109/TCSI.2009.2019389
[11] B. Lincoln and A. Rantzer, *Relaxing dynamic programming*, IEEE Trans. Automat. Contr. 51 (8) (August 2006), pp. 1249–1260. doi:10.1109/TAC.2006.878720
[12] M. Santos, *Chapter 2: Modelling switched power converters using the complementarity formalism*, doctoral thesis, Universitat Politècnica Catalunya, December 2006.
[13] F. Vasca, L. Iannelli, M. Kanat Camlibel, and R. Frasca, A new perspective for modeling power electronics converters: Complementarity framework, IEEE Trans. Power Electron. 24 (2) (February 2009), pp. 456–468. doi:10.1109/TPEL.2008.2007420
[14] R.-J. Wai and L.-C. Shih, Design of voltage tracking control for DC–DC boost converter via total sliding-mode technique, IEEE Trans. Ind. Electron. 58 (6) (June 2011), pp. 2502–2511. doi:10.1109/TIE.2010.2066539
[15] E. Vidal-Idiarte, C.E. Carrejo, J. Calvente, and L. Martinez-Salamero, Two-loop digital sliding mode control of DC–DC power converters based on predictive interpolation, IEEE Trans. Ind. Electron. 58 (6) (June 2011), pp. 2491–2501. doi:10.1109/TIE.2010.2069071
[16] R. Baždarić, I. Škrjanc, and D. Matko, *Two degrees of freedom in the control of a DC-DC boost converter, fuzzy identified explicit model in feed-forward line*, J. Intell. Robotic Syst.. Springer Netherlands. 2015. ISSN 0921-0296. doi:10.1007/s10846-015-0215-1
[17] T. Geyer, G. Papafotiou, and M. Morari, Hybrid model predictive control of the step-down DC–DC converter, IEEE Trans. Control Syst. Technol. 16 (6) (November 2008), pp. 1112–1124. doi:10.1109/TCST.2008.917221
[18] J.M. Maciejowski, *Predictive Control*, Prentice- Hall, Englewood Cliffs, NJ, 2002.
[19] A.A. Stoorvogel and T.J.J. van den Boom, *Model predictive control*, DISC Course Lecture Notes, DISC Delft, The Netherlands, January 2010.
[20] M.V. Kothare, V. Balakrishnan, and M. Morari, *Robust constrained model predictive control using linear matrix inequalities*, Automatica. 32 (1996), pp. 1361–1379. Elsevier Science. doi:10.1016/0005-1098(96)00063-5
[21] T. Takagi and M. Sugeno, *Fuzzy identification of systems and its applications to modeling and control*, IEEE Trans. Syst. Man Cybern. SMC-15 (1) (1985), pp. 116–132. doi:10.1109/TSMC.1985.6313399
[22] S. El Beid and S. Doubabi, *DSP-based implementation of fuzzy output tracking control for a boost converter*, IEEE Trans. Ind. Electron. Regular Pap. 61 (1) (January 2014), pp. 196–209. doi:10.1109/TIE.2013.2242413

[23] R.D. Middlebrook, *Small-signal modeling of pulse-width modulated switched-mode power converters*, Proc. IEEE. 76 (4) (April 1988), pp. 343–354. doi:10.1109/5.4421

[24] R.W. Erickson, S. Ćuk, and R.D. Middlebrook, *Large-signal modelling and analysis of switching regulators*, In: IEEE, 1982 PESC Rec., Cambridge, MA, pp. 240–250.

[25] W.P.M.H. Heemels, B. De Schutter, and A. Bemporad, *On the equivalence of classes of hybrid dynamical models*, Proceedings of the 40th IEEE Conference on Decision and Control, Orlando, Florida, pp. 364–369, December. 2001.

[26] A. Bemporad and M. Morari, *Control of systems integrating logic, dynamics, and constraints*, Automatica 35 (1999), pp. 407–427. doi:10.1016/S0005-1098(98)00178-2

[27] A.J. Van Der Schaft and J.M. Schumacher, *Complementarity modeling of hybrid systems*, IEEE Trans. Automat. Contr. 43 (4) (April 1998), pp. 483–490. doi:10.1109/9.664151

[28] C. Goffman and G. Pedrick, *First Course in Functional Analysis*, Prentice-Hall Inc., Englewood Cliffs, NJ, 1965.

[29] The MathWorks Inc, *"MATLAB", R2009a, 7.8.0.347*, Natick, MA, February 2009.

[30] J. Sjöberg, Q. Zhang, L. Ljung, A. Benveniste, B. Delyon, P.-Y. Glorennec, H. Hjalmarsson, and A. Juditsky, Nonlinear black-box modeling in system identification: a unified overview, Automatica 31 (12) (1995), pp. 1691–1724. doi:10.1016/0005-1098(95)00120-8

[31] D. Matko and I. Skrjanc, *Predictive functional control based on fuzzy model for heat-exchanger pilot plant*, IEEE Trans. Fuzzy Syst. 8 (6) (2000), pp. 705–712. doi:10.1109/91.890329

[32] K. Kavsek-Biasizzo, I. Škrjanc, and D. Matko, *Fuzzy predictive control of highly nonlinear pH process*, Comput. Chem. Eng. 21 (1–2) (1997), pp. S613–S618. doi:10.1016/S0098-1354(97)00117-8

[33] I. Škrjanc, S. Blažič, S. Oblak, and J. Richalet, *An approach to predictive control of multivariable time-delayed plant: Stability and design issues*, ISA Trans. 43 (2004), pp. 585–595. doi:10.1016/S0019-0578(07)60170-0

[34] J. Castro, *Fuzzy logic controllers are universal approximators*, IEEE Trans. Syst. Man Cybern 25 (1995), pp. 629–635. doi:10.1109/21.370193

[35] L.-X. Wang, Design and analysis of fuzzy identifiers of nonlinear dynamic systems, IEEE Trans. Automat. Contr. 40 (1995), pp. 11–23. doi:10.1109/9.362903

[36] K.M. Passino and S. Yurkovich, *Fuzzy Control*, Addison Wesley Longman, Inc., Menlo Park, CA, 1998.

[37] P. Karamanakos, G. Papafotiou, and S.N. Manias, *Model predictive control of the interleaved DC-DC boost converter*, 15th International Conference on System Theory, Control, and Computing (ICSTCC), Sinaia, Romania, 2011.

[38] K.S. Holkar and L.M. Waghmare, *An overview of model predictive control*, Int. J. Control Automat. 3, 4 (December 2010), pp. 47–63.

[39] H.B. Kuntze, A. Jacubasch, J. Richalet, and C. Arber, *On the predictive functional control of an elastic industrial robot*, Proceedings of 25th Conference on Decision and Control, Athens, Greece, December 1986.

[40] S. Abu El Ata-Doss, P. Fiani, and J. Richalet, *Handling input and state constraints in predictive functional control*, Proceedings of 30th Conference on Decision and Control, Brighton, England, December 1991.

[41] I. Škrjanc and D. Matko, *Fuzzy predictive functional control in the state space domain*, J. Intell. Robotic Syst. 31 (2001), pp. 283–297. doi:10.1023/A:1012011010623